Regret Minimization in MDPs with Options


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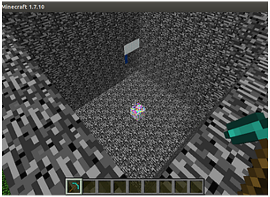
In this talk: **on-line learning** with options

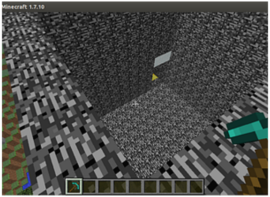
Example: Minecraft [?]

- Three subtasks

Navigate



Pick-up



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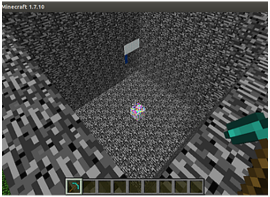

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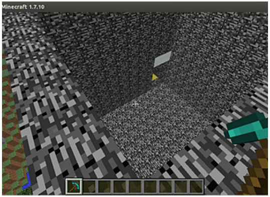

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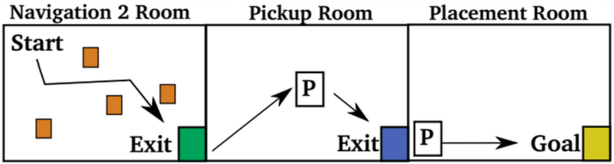
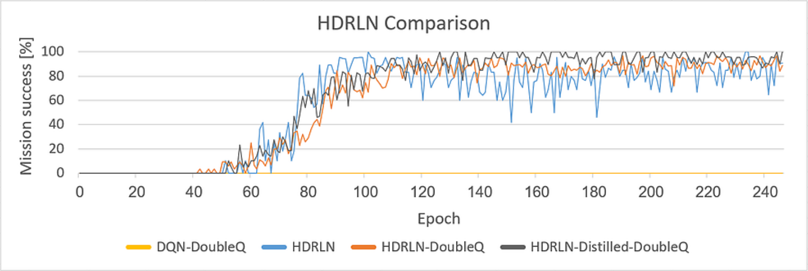
- One macro-task combining all three subtasks



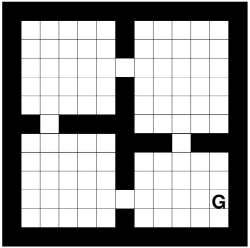
Example: Minecraft [?]

- Simulations



Options Limitations

Four-rooms maze [?]



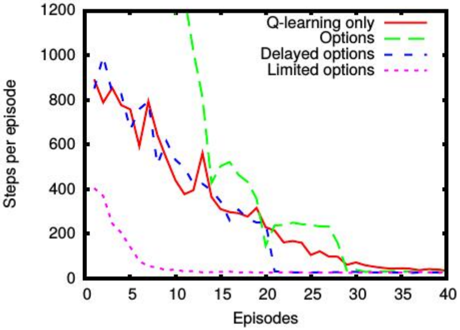


Figure 4: Comparison of learning agents with varying access to correct temporal abstractions. The Q-learning agent never uses options. The "options" agent gains immediate access to the correct options everywhere. The "delayed options" agent gains access to these options after 20 episodes. The "limited options" agent gains immediate access to these options except in the lower-right room. Each learning curve is the average of 50 independent runs.

Research questions

Empirical observations: introducing options in an MDP can *speed up* learning [**?**] but can also be *harmful* [**?**].

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Option Design: a challenging problem that has been always empirically tackled:

- Leveraging on MDP properties (e.g., bottleneck discovery [**?**], Laplacian analysis [**?**])
- Direct option optimization (e.g., Option-Critic [**?**])
- and many other concepts

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$\rightsquigarrow$ Can we exploit theoretical *properties to design options*?

Disclaimer:

this talk is not about option design

options are **assumed** to be given as input

Problem addressed in this talk

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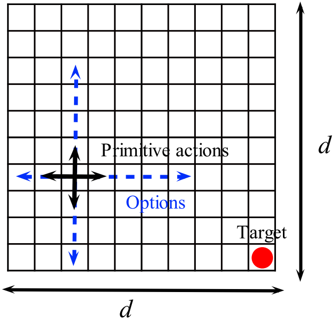

GRID WORLD

Replace all 4 cardinal actions by *cardinal options*:

- Same number of states
- Same number of actions

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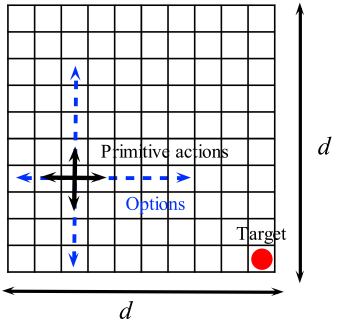

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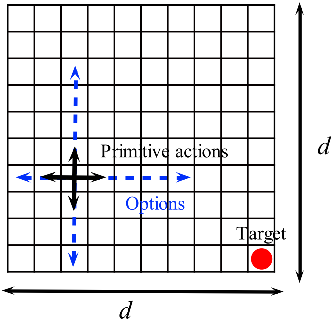

<u>Grid world</u>

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Question: What is the impact of options in the above example? How is exploration affected by options?

Markov Decision Processes

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, p, r\}$$

- $\mathcal{S}$ is the state space
- $\mathcal{A} = (\mathcal{A}_s)_{s \in \mathcal{S}}$ is the set of actions
- when choosing action a in state s:
  - random reward with mean $r(s, a) \in [0, 1]$
  - transition to the next state $(s, a) \to s'$ according to transition probability distribution $p(\cdot|s, a)$

[?]

Average Reward MDP

Policies, gain and optimality

The average expected reward (or **gain**) of a policy π is

$$g(\mathcal{M},\pi) = \lim_{N\to\infty} \frac{1}{N}\mathbb{E}\left[\sum_{t=1}^{N} r(s_t,a_t)|\mathcal{M},\pi\right]$$

where $a_t \sim \pi(\cdot|s_t)$.

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Learner's Goals:

1. Find the *optimal* policy $\pi^* = \arg\max_{\pi} g(\mathcal{M}, \pi)$

OPTIMALITY EQUATION

$$g^* = \max_{a} \left\{ r(s,a) + p^T u^* - u^*(s) \right\}$$

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$\rightsquigarrow$ **Regret minimization!**

FREQUENTIST REGRET: *optimism in face of uncertainty (OFU)*

$$\Delta(\mathcal{M},\mathfrak{A},T) = Tg^*(\mathcal{M}) - \sum_{t=1}^{T} r_t$$

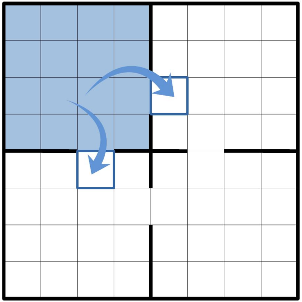
Temporal Abstraction

The Option Framework

Definition 1

A (Markov) Option o is a 3-tuple $\{s_o, \beta_o, \pi_o\}$ where:

- $s_o \in \mathcal{S}$ is the **states** where the option can be initiated,
- $\beta_o : \mathcal{S} \to [0, 1]$ is a **Markov termination condition**,
- $\pi_o \in \Pi_M^{SR}$ is a **stationary Markov policy**.



Temporal Abstraction

Semi-Markov Decision Processes

SMDP [?]

A set of options $\mathcal{O}$ defined on an MDP M induces an SMDP M' :

$$M + \mathcal{O} \implies M'$$

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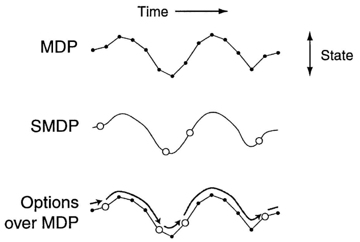


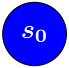
A Semi-Markov Decision Process

$$M' = \{\mathcal{S}', \mathcal{A}', p, r, \tau\}$$

is an MDP with a **random holding time** $\tau(s,a)$ associated with any state-action pair.

Learning in SMDP

T_n: number of time steps
n: number of decision steps



$n = 0 \qquad T_0 = 0$

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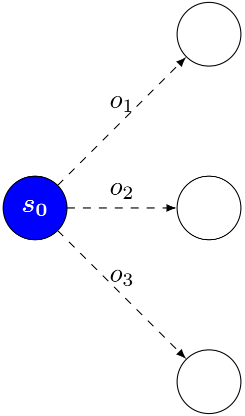

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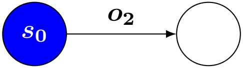

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$$s_0 \xrightarrow{\boldsymbol{\tau_1},\ \boldsymbol{R_1}} \boldsymbol{s_1}$$

$$n = 1 \qquad T_1 \leftarrow T_0 + \tau_1 \qquad R_1 \leftarrow \sum_{t=1}^{\tau_1} r_t$$

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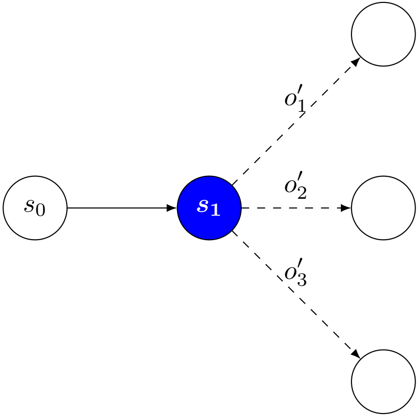

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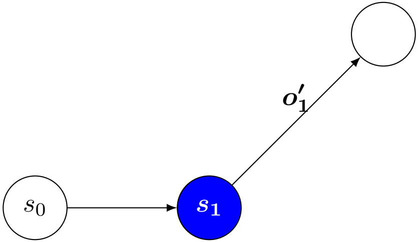

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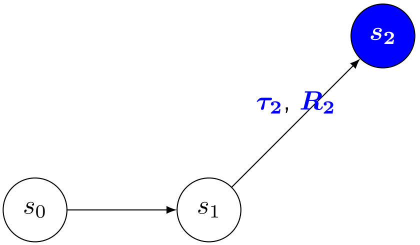

$$n = 2 \qquad T_2 \leftarrow T_1 + \tau_2 \qquad R_2 \leftarrow \sum_{t=1}^{\tau_2} r_t$$

$$\implies T_n = \sum_{i=1}^{n} \tau_i$$

Optimal Policy and Regret

Optimal policy

$$g^*_{\mathcal{O}} = \max_{\pi} g^{\pi}_{\mathcal{O}} = \max_{\pi} \lim_{n\to\infty} \mathbb{E}^{\pi}\left[\frac{\sum_{t=1}^{n} R_t}{T_n}\right]$$

$$\pi^* : \mathcal{S} \to \mathcal{O}$$

Frequentist (SMDP) regret

$$\Delta(\mathcal{M}, \mathfrak{A}, T_n) = T_n g^*_{\mathcal{O}} - \sum_{i=1}^{n} R_i$$

SMDP-UCRL

1. Construct set of *plausible SMDPs* $\mathcal{M}_k = \Phi(\mathcal{H}_k)$
   By exploiting confidence interval on
   - Option transition probability: $\beta_k^p(s, o)$
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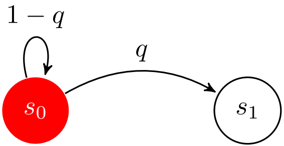
EXAMPLE



$$\mathbb{E}[\tau] = \frac{1}{q}$$

$$P(\tau = k) = (1-q)^{k-1} q$$

What about a confidence interval on τ?

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[?, Lemma 3]

If the set of options is proper, all **holding times and rewards are sub-Exponential**. Moreover, they are sub-Gaussian if and only if they are bounded.

This property comes from its inner Markov structure
(to be continued)

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2. Compute $\pi_k \in \arg\max_{\pi \in \Pi_{\mathcal{O}}, M \in \mathcal{M}_k} g_{\mathcal{O}}(M, \pi)$
   - Use EVI to solve the optimality equation

$$\tilde{g}_{\mathcal{O}}^* = \max_{o \in \mathcal{O}_s} \left\{ \max_{R \in \widetilde{\boldsymbol{R}}, \tau \in \widetilde{\boldsymbol{\tau}}} \left\{ \frac{R(s,o)}{\tau(s,o)} + \frac{1}{\tau(s,o)} \left(\max_{p \in \widetilde{\boldsymbol{p}}(s,o)} \left\{ p^T u^* \right\} - u^*(s) \right) \right\} \right\}$$

Regret for MDPs

Theorem 2

In a finite MDP with diameter D, with probability at least $1-\delta$ the regret of UCRL after T_n time steps is bounded by

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Diameter

The *diameter* of an MDP $\mathcal{M}$ is the maximal expected time it takes to reach any state from any other state under an appropriate policy

$$D(\mathcal{M}) := \max_{s,s' \in \mathcal{S}, s \neq s'} \min_{\pi} \boxed{\mathbb{E}^{\pi}\left[T(s')|s_0 = s\right]}$$

Mean first passage time

Def. *Communicating MDP* $\Leftrightarrow$ *Finite Diameter*

Regret analysis for SMDPs

[?, Theorem 1]

High-probability regret bound for SMDP-UCRL in M':

$$\Delta(M', \mathfrak{A}, T_n) = O\left(\left(D'\sqrt{S'} + \boldsymbol{\mathcal{C}(M', n, \delta)}\right)\sqrt{S'A'n\log\left(\frac{n}{\delta}\right)}\right)$$

where $\mathcal{C}(M', n, \delta)$ is

h.p. bound on holding time

$$\mathcal{C}(M', n, \delta) = \tau_{\max} + C_\tau\sqrt{\log\left(\frac{n}{\delta}\right)}$$

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Comparing regrets SMDP/MDP (ratio):

$$\mathcal{R} \sim \frac{D'}{D}\sqrt{\frac{On}{AT_n}}$$

Comments on SUCRL

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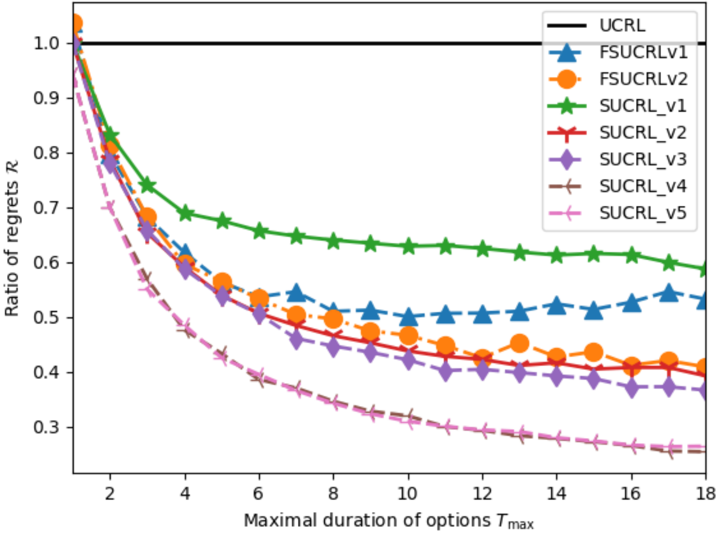
- Avoid considering options as atomic operations
- Take into account the inner option MDP structure

Experiments

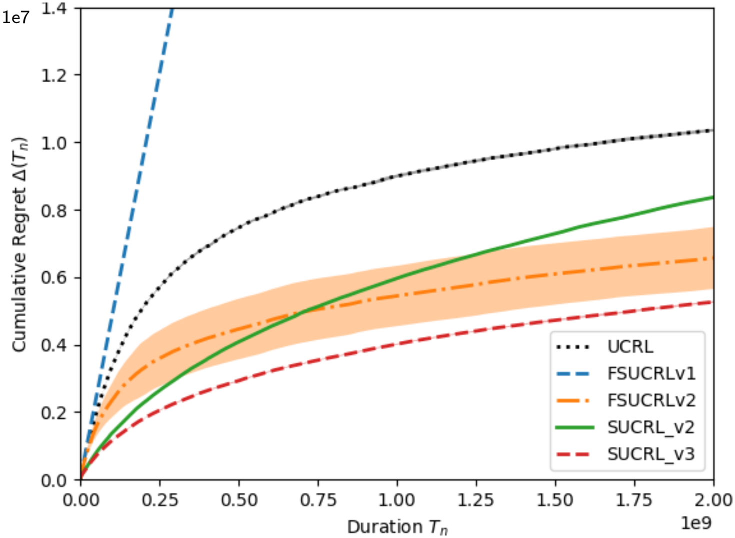
Grid World

Domain presented in the introduction



Four Rooms 14x14

The classical domain for options



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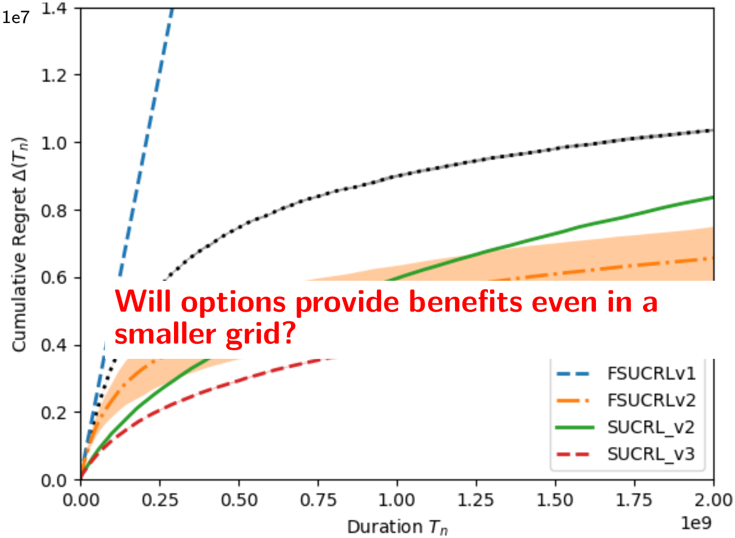

Will options provide benefits even in a smaller grid?

Four Rooms 6x6

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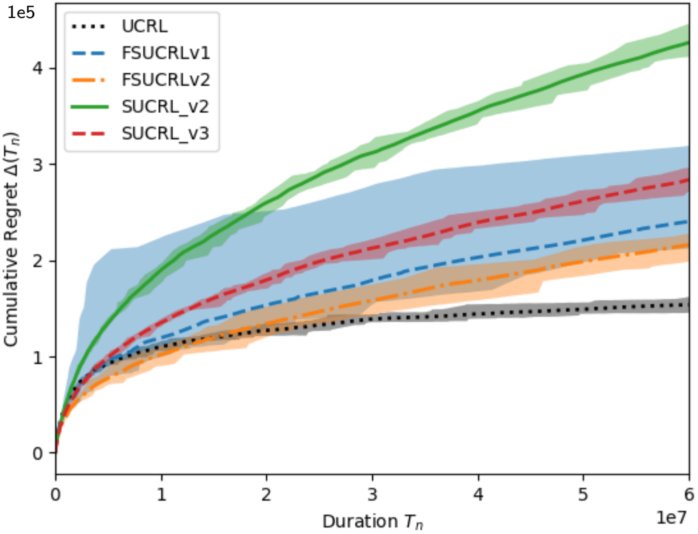

Conclusions

- Temporal abstraction is **powerful**
  - Faster learning
  - Less regret
- But it does not come for free
  - May increase the computational complexity
  - Requires a far-sighted design of options

Thank you for your attention

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